NAME: KEY DATE: 12/7/2016 PERIOD:__________ PHYSICS

- Review Sheet – 2-D Kinematics & Projectile Motion -

Students should know or be able to do the following:

2-D Kinematics

1. Students should know that motion is defined relative to some point in space.
2. Students should understand how time can be utilized in both the x- and y-dimensions.
3. Students should be able to solve 2-D kinematics problems for an unknown variable.

Horizontal and Ground Launched Projectiles

1. Students should know that projectiles follow a parabolic shaped path.
2. Students should be able to determine the components of velocity at any time in flight.
3. Students should understand the relationship between the following variables:
   a. time of flight, v_{iy}, d_y and a
   b. time of flight, v_{ix}, and d_x.
4. Students should know that, for a horizontal projectile, the following assumptions apply:

   X – direction
   i. V_x = # Given
   ii. $a = 0\ m/s^2$ ***CONSTANT VELOCITY***
   iii. Use constant velocity equations → $v = d / t$

   y – direction
   i. $Vi_y = 0\ m/s$
   ii. object is in freefall, so $a = -9.8\ m/s^2$
   iii. Use kinematic equations
5. Students should know that, for a ground launched projectile, the following assumptions apply:

   x-direction
   i. initial velocity ($v_{ix} = v \cos \theta$)
   ii. $a = 0\ m/s^2$ ***CONSTANT VELOCITY***
   iii. horizontal velocity is CONSTANT!
   iv. USE CONS. V equation to solve.

   y-direction
   i. initial velocity ($v_{iy} = v \sin \theta$)
   ii. acceleration due to gravity = $-9.8\ m/s^2$
   iii. $v_{fy} = 0$ (at the top of parabolic path, or max height)
   iv. Total time = 2* time to top
   v. Treat as a FREEFALL problem in vertical direction...
   vi. USE KINEMATIC EQ's to solve.

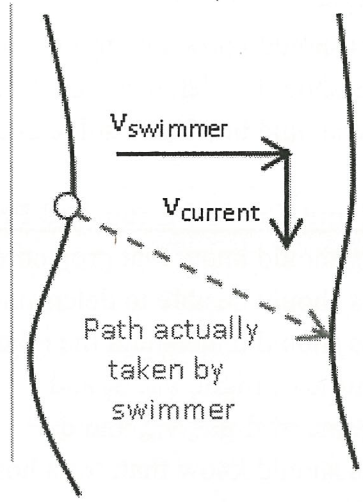
6. Students should be able to calculate unknown variables using kinematic equations in two directions.

2-D Kinematics

horizontal: v_{const} ($a_x = 0$)
vertical: v_{const} ($a_y = 0$)

Path will be straight

ex. River prob.



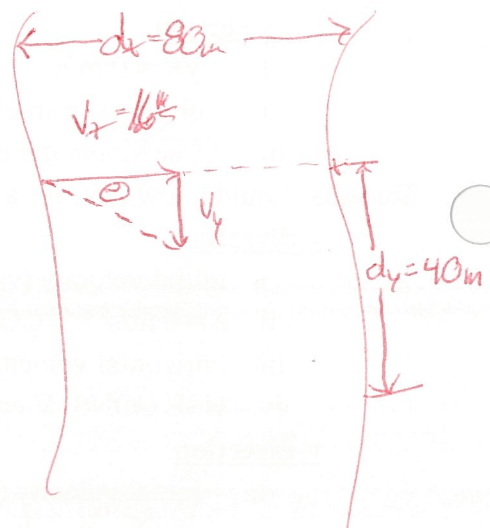
Problem Solving Strategy:

1. Start with a sketch.
2. Draw / label all given information
3. Create an "x / y" table, and put information on correct side
   a. Horizontal → x direction
   b. Vertical → y direction
4. Solve for desired quantity

****NOTE**** - Time is the ***ONE*** variable that can be used in ***BOTH*** directions.

Practice Problems

1. A swimmer heads directly across a river swimming at 1.6 m/s relative to still water. She arrives at a point 40 m downstream from the point directly across the river, which is 80 m wide. Determine…



a. the speed of the current

$d_y = 40m$
$t = 50s$

$V_y = \frac{d_y}{t} = \frac{40m}{50s}$

$V_y = 0.8 \frac{m}{s}$

b. the magnitude of the swimmer's resultant velocity

$V_R = \sqrt{V_x^2 + V_y^2} = \sqrt{(1.6\frac{m}{s})^2 + (0.8\frac{m}{s})^2}$

$V_R = 1.79 \frac{m}{s}$

c. the direction of the swimmer's resultant velocity

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{0.8\frac{m}{s}}{1.6\frac{m}{s}}\right)$

$\theta = 26.6°$

d. the time it takes the swimmer to cross the river

$d_x = 80m$
$V_x = 1.6 m/s$

$t = ?$

$V_x = \frac{d_x}{t} \Rightarrow t = \frac{d_x}{V_x} = \frac{80m}{1.6\frac{m}{s}}$

$t = 50s$

2. A plane flies due north at a speed of 200 m/s while encountering a ***crosswind*** of 90 m/s to the east. The plane travels for 30 minutes. Determine…

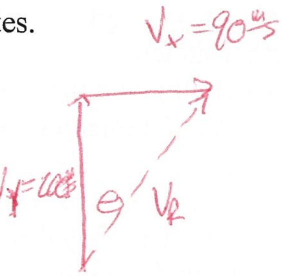
a. the total distance east that the plane has traveled at the end of 30 minutes.

$t = 30\,\text{min}\left(\frac{60\,s}{1\,\text{min}}\right) = 1800\,s$

$V_x = 90\,\frac{m}{s}$

$d_x = V_x \cdot t = 90\,\frac{m}{s} \cdot 1800\,s$

$d_x = 162{,}000\,m$



b. the total distance north that the plane has traveled at the end of 30 minutes.

$t = 1800\,s$

$V_y = 200\,\frac{m}{s}$

$d_y = V_y \cdot t = 1800\,s \cdot 200\,\frac{m}{s}$

$d_y = 360{,}000\,m$

c. the magnitude of the planes resultant velocity

$V_R = \sqrt{V_x^2 + V_y^2} = \sqrt{(90\,\frac{m}{s})^2 + (200\,\frac{m}{s})^2}$

$V_R = 219.3\,\frac{m}{s}$

d. the direction of the planes resultant velocity

$\theta = \tan^{-1}\left(\frac{V_x}{V_y}\right) = \tan^{-1}\left(\frac{90\,\frac{m}{s}}{200\,\frac{m}{s}}\right)$

$\theta = 24.2°$

Projectile Motion – Horizontal & Ground Launched

Problem Solving Strategy:
1. Start with a sketch.
2. Draw / label all given information
3. Create an "x / y" table, and put information on correct side
   a. Horizontal → x direction
   b. Vertical → y direction
4. Solve for desired quantity

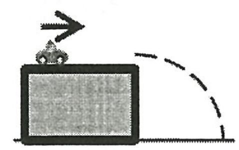
*****NOTE***** - Time is the ***ONE*** variable that can be used in ***BOTH*** directions.

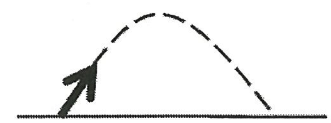
horizontal: v_{const} ($a_x = 0$)
vertical: a_{const} ($a_y = -9.81\ m/s^2$)

Represents parabolic path

ex. Projectiles

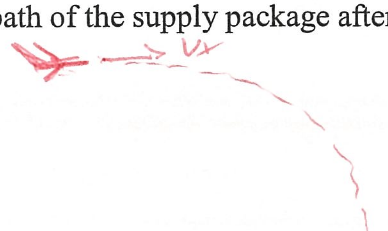
Horizontal Projectile	Ground Launched Projectile
$v_{iy} = 0$ $a_x = 0$ $a_y = 9.81\ m/s^2$	$v_{iy} = v_i \sin\theta$ $v_{ix} = v_i \cos\theta$ $a_x = 0$ $a_y = 9.81\ m/s^2$ $t_{top} = 1/2\ t_{tot}$





Practice Problems

1. A supply plane flying **horizontally** at a speed of 250 m/s east at an altitude of 490 meters drops a supply package to a crew on the ground. It falls freely without a parachute.
   a. Sketch a path of the supply package after it leaves the plane.



   b. Determine the time required for the package to reach the ground.

$V_{iy} = 0\ m/s$
$d = 490\ m$
$a = -9.8\ m/s^2$

$d = \cancel{V_i t}^{0} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(490\ m)}{-9.8\ m/s^2}} = 10\ s$

   c. Determine the horizontal distance from the target that the plane must drop the package.

$V_x = 250\ m/s$
$t = 10\ s$

$d_x = V_x \cdot t = (250\ m/s)(10\ s)$
$d_x = 2,500\ m$

   d. Determine the horizontal and vertical components of the package's velocity right before the package reaches the ground.

$V_x = 250\ m/s$

$V_{Fy} = \cancel{V_i}^{0} + at$
$V_{Fy} = -9.8\ m/s^2 \cdot 10\ s$
$V_{Fy} = -98\ m/s$

2. A player kicks a football from ground level at 24.0 m/s at an angle of 35.0 degrees above the horizontal.
   a. Calculate the initial horizontal and vertical components of the football's velocity.

$V_x = V\cos\theta = 24\frac{m}{s}\cos 35^\circ$ $\quad V_y = V\sin\theta = 24\frac{m}{s}\sin 35^\circ$

$V_x = 19.7\frac{m}{s}$ $\quad V_y = 13.8\frac{m}{s}$

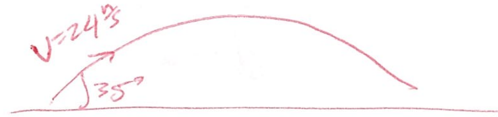
   b. Calculate the maximum height that the football reaches above the ground.

$V_{iy} = 13.8\frac{m}{s}$ $\quad V_F^2 = V_i^2 + 2ad$

$V_F = 0\frac{m}{s}$

$a = -9.8\frac{m}{s^2}$ $\quad d = \frac{-V_i^2}{2a} = \frac{-(13.8\frac{m}{s})^2}{2(-9.8\frac{m}{s^2})}$

$d = 9.7m$



   c. Calculate the total amount of time that the football is in the air for.

$V_{Fy} = 0\frac{m}{s}$ $\quad V_F = V_i + at$ $\quad t_{TOT} = 2 \cdot t_{up} = 2 \cdot 1.41s$

$V_{iy} = 13.8\frac{m}{s}$ $\quad t_{up} = \frac{-V_i}{a} = \frac{-13.8\frac{m}{s}}{-9.8\frac{m}{s^2}}$ $\quad t_{TOT} = 2.82s$

$a = -9.8\frac{m}{s^2}$

$t_{up} = 1.41s$

   d. Calculate the total horizontal distance that the football travels.

$V_x = 19.7\frac{m}{s}$ $\quad d_x = V_x \cdot t = 19.7\frac{m}{s} \cdot 2.82s$

$t_{TOT} = 2.82s$ $\quad d_x = 55.6m$

3. During a world record attempt to jump over a whole football field, Travis Pastrana rides his dirt bike at 35 m/s off a ramp that makes an angle of 45° to the horizontal.
   a. Find the maximum height that Travis reaches during this world record attempt.

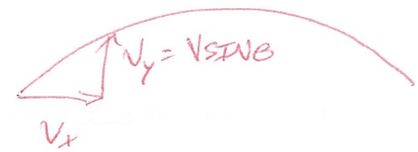
$V_{iy} = 35\frac{m}{s} \cdot \sin 45 = 24.7\frac{m}{s}$ $\quad V_F^2 = V_i^2 + 2ad$

$V_F = 0\frac{m}{s}$ $\quad d = \frac{-V_i^2}{2a} = \frac{-(24.7\frac{m}{s})^2}{2(-9.8\frac{m}{s^2})} = 31.1m$

$a = -9.8\frac{m}{s^2}$

   b. What is the total amount of time that Travis spends in the air?

$V_i = 24.7\frac{m}{s}$ $\quad V_F = V_i + a\,t_{up}$



$V_F = 0\frac{m}{s}$ $\quad t_{up} = \frac{-V_i}{a} = \frac{-24.7\frac{m}{s}}{-9.8\frac{m}{s^2}} = 2.52s$

$a = -9.8\frac{m}{s^2}$ $\quad t_{TOT} = 2 \cdot t_{up} = 5.04s$

   c. What is the total range that Travis travels during his flight?

$V_x = V\cos\theta = 35\frac{m}{s}\cos 45^\circ = 24.7\frac{m}{s}$ $\quad V_x = \frac{d_x}{t} \Rightarrow d_x = V_x \cdot t = 24.7\frac{m}{s} \cdot 5.04s$

$t_{TOT} = 5.04s$ $\quad d_x = 124.5m$

4. A toy car is rolled off the edge of a 1m high table with a speed of 7 m/s.

a. What is the vertical velocity of the car the instant before it hits the ground.

$V_{iy} = 0 \frac{m}{s}$
$a = -9.8 \frac{m}{s^2}$
$d = 1m$

$V_F^2 = V_i^2 + 2ad$
$V_F = \sqrt{2ad} = \sqrt{2(-9.8 \frac{m}{s^2})(-1m)}$
$V_F = -4.4 \frac{m}{s}$

b. How long does it take for the car to reach the ground?

$V_{iy} = 0 \frac{m}{s}$
$a = -9.8 \frac{m}{s^2}$
$d = 1m$

$d = V_i t + \frac{1}{2} a t^2$
$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-1m)}{-9.8 \frac{m}{s^2}}}$
$t = 0.45s$

c. How far from the base of the table does the car land?

$V_x = 7 \frac{m}{s}$
$t = 0.45s$

$d_x = V_x \cdot t = 7 \frac{m}{s} \cdot 0.45s$
$d_x = 3.15m$

5. A snowboarder leaves a ramp at an angle of 22° to the horizontal with a speed of 15.0 m/s.

a. Find the maximum height that the snowboarder reaches.

$V_{iy} = V \sin\theta = 15 \frac{m}{s} \sin 22° = 5.6 \frac{m}{s}$
$V_{Fy} = 0 \frac{m}{s}$
$d = ?$
$a = -9.8 \frac{m}{s^2}$

$V_F^2 = V_i^2 + 2ad$
$d = \frac{-V_i^2}{2a} = \frac{-(5.6 \frac{m}{s})^2}{2(-9.8 \frac{m}{s^2})}$
$d = 1.6m$

b. What is the total amount of time that the snow boarder spends in the air?

$V_{iy} = 5.6 \frac{m}{s}$
$V_{Fy} = 0 \frac{m}{s}$
$a = -9.8 \frac{m}{s^2}$

$t_{TOT} = ?$

$V_F = V_i + a t_{up}$
$t_{up} = \frac{-V_i}{a} = \frac{-5.6 \frac{m}{s}}{-9.8 \frac{m}{s^2}}$
$t_{up} = 0.57s$

$t_{TOT} = 2 \cdot t_{up}$
$t_{TOT} = 1.14s$

c. How far in the horizontal directions from the base of the ramp does the snowboarder land??

$V_x = V\cos\theta = 15 \frac{m}{s} \cos 22° = 13.9 \frac{m}{s}$
$t_{TOT} = 1.14s$

$d_x = V_x \cdot t = 13.9 \frac{m}{s} \cdot 1.14s$
$d_x = 15.8m$

d. What is the ***total*** velocity of the snowboarder at maximum height?

$V_x = 13.9 \frac{m}{s}, \; V_y = 0 \frac{m}{s}$

e. What is the acceleration of the snowboarder at maximum height?

$a = -9.8 \frac{m}{s^2}$